

Modifying degrees and their proportions

Andrew Kato
 UC Santa Cruz
 anmkato@ucsc.edu

S-Circle, Winter 2025

1. Introduction

It's well known since [Westerståhl \(1985:403\)](#) that the vague quantifiers *many/few* can be three-way ambiguous between cardinal, proportional, and relative proportional readings ([Partee 1989](#); [Herburger 1997](#)), given a contextual threshold (n):

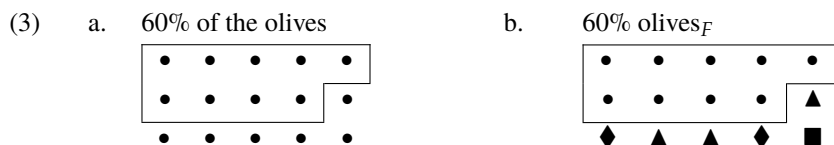
- (1) a. Many [_S Scandinavians] [_N have won the Nobel Prize in literature].
 $\rightsquigarrow |S \cap N| \geq n$ (cardinal)
 $\rightsquigarrow |S \cap N|/|S| \geq n$ (proportional)
 b. Many [_S Scandinavians_F] [_N have won the Nobel Prize in literature].
 $\rightsquigarrow |S \cap N|/|N| \geq n$ (relative proportional)

More recently, it's observed that *precise* proportional quantifiers (relative measures, RMs) are also amenable to a non-conservative reading (following [Ahn & Sauerland 2015a,b, 2017](#)):

- (2) a. The fruit supplier sold 60% of the olives.
 b. The fruit supplier sold 60% olives_F.

With the removal of (overt) partitive material and the addition of focus-marking on the substance noun (*olives*), (2b) yields a high-scope reading.

Descriptively, (2b) asserts that, out of all things a particular fruit supplier bought (restrictor), 60% are olives (scope). We might visualize the state of affairs of one sentence versus the other as follows:



The high-level focus of this talk is to look at what happens when we modify RM expressions (4). We will consider interactions between non-conservative RMs and numeral modifiers, particularly those which may implicate other degree-related, scope-bearing elements.

- (4) a. The university accepted between 20 and 30% transfer_F students.
 b. The veterinarian's office saw up to 20% dogs_F in their weekly appointments.
 c. The soda contains as much as 40% sugar_F.

The main puzzle under discussion is the ability for a modified non-conservative RM to participate in cumulative readings — under which (5) is true just in case ...

- The total number of interviewing recruiters is 2, and
- The total proportion of women interviewed by recruiters, out of all interviewees, is 60%.

- (5) [Exactly two recruiters] interviewed [exactly 60% women_F].

We'll draw on an ontology of negative entities ([Bledin 2024](#); [Elliott 2024](#)) to enforce the cumulative reading. This will allow us to preserve an intuitively scope-taking account of RM phrases as degree quantifiers, while also ruling out pseudo-cumulativity. Four points will be covered along the way:

- *Point 1*: Can a degree-quantificational semantics for RMs get us a cumulative reading?
 \rightsquigarrow Yes! (by way of 'polarizing' entities, [Bledin 2024](#); [Elliott 2024](#))
- *Point 2*: Can the same semantics also get us narrow scope for unmodified RMs?
 \rightsquigarrow Yes! (by way of degree pluralities, [Dotlačil & Nouwen 2016](#); [Li 2022](#))
- *Point 3*: Will RMs without non-monotone modifiers be accounted for too?
 \rightsquigarrow Yes! (we'll look at quantity equatives, [Rett 2015](#))
- *Point 4*: Do we cover empirical ground by polarizing degree pluralities too?
 \rightsquigarrow No! (I suggest that degree negation may be problematic to implement)

The primary items are Points 1 and 2, which we proceed to tackle first.

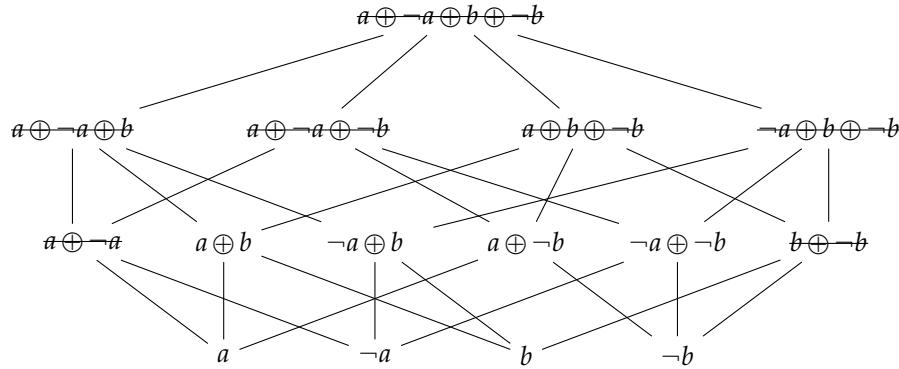


Figure 1: Summation over four atoms — $\otimes\{a, \neg a, b, \neg b\}$

2. Negation and pluralities

Bledin (2024) observes that some expressions intuitively seem to express individual exclusion or non-participation (6). Accordingly, this is used as motivation to implement the availability of negation at the level of entities and their pluralities.

- (6) a. $\llbracket[\text{Not Anne but Brooke}]\rrbracket \approx \neg\text{Anne} \oplus \text{Brooke}$
 b. $\llbracket[\text{Neither New York nor Los Angeles}]\rrbracket \approx \neg\text{NY} \oplus \neg\text{LA}$
 c. $\llbracket[\text{Ringo and no one else}]\rrbracket \approx \text{Ringo} \oplus \neg\text{Paul} \oplus \neg\text{John} \oplus \dots$

While I won't spend time recapitulating arguments for the existence of entity negation *per se*, adopting this ontological upgrade will be useful for specifying the *absence* of individuals.¹

Elliott (2024) picks up this proposal to develop a candidate theory of modified numerals and the semantics of *zero*, based on enriched sets of pluralities.

Extending his analysis, I start by setting up a revised version of predicate cumulation (7), assuming that entity negation applies over all atoms of the domain of entities D_e .²

- (7) a. $*P = \min_{\subseteq} \{P' \mid P \subseteq P' \wedge \forall x, y \in P' [x \oplus y \in P']\}$
 b. $\otimes P = *P \setminus \{X \in *P \mid \exists x \in X [\neg x \in X]\}$

Figure 1 depicts such an example. We obtain the smallest sum-closed superset of P , minus

¹We may conceptualize entity negation as something akin to exact falsification for entities, in the context of a truthmaker semantics (Fine 2017, et seq.; Elliott 2020; Moltmann 2020). For Bledin's (2024) unilateral system, it is roughly a means of tracking participation in eventualities through the compositional process.

²(7a) is one of multiple ways to define Link's (1983) star operator (Champollion & Krifka 2016).

combinations including an atom and its negative counterpart ('incoherence', Elliott 2024).

It follows that the conventional denotation of a plural noun, e.g., (8), will include negative, non-negative, and mixed polarity sums.³ If parthood (subsethood) and summation (union) are themselves set-based, $|\cdot|^+$ counts non-negative atoms (e.g., $|x|^+ = 2$ for $x = a \oplus b \oplus \neg c$).

- (8) a. $\llbracket[\text{olives}]\rrbracket = \lambda x. \otimes \text{olive } x$ Se
 b. $\llbracket[\text{one olive}]\rrbracket = \lambda x. \otimes \text{olive } x \wedge |x|^+ \geq 1 \rightsquigarrow \{a, b, c, a \oplus b, a \oplus \neg b, \dots\}$
 c. $\llbracket[\text{two olives}]\rrbracket = \lambda x. \otimes \text{olive } x \wedge |x|^+ \geq 2 \rightsquigarrow \{a \oplus b, \dots, a \oplus b \oplus \neg c, \dots\}$

Composing a mixed-polarity predicate with a numeral n gives the set of relevant entities with at least n -many non-negative atoms.

Since predicates will generally no longer have unique maximal pluralities (recall Fig. 1), the parthood maximum operator \max_{\subseteq} yields a set (9).

- (9) a. $\max_{\subseteq} := \lambda P \lambda x. P x \wedge \neg \exists y [P y \wedge x \sqsubset y]$ Se \rightarrow Se
 b. $\max_{\subseteq}(\{a, \neg a\}) = \{a, \neg a\}$ (multiple parthood maxima)

3. Interpreting proportions

With a basic sketch of pluralities, we can return to a semantics for proportions. The broad consensus is that RM nouns such as *percent* (10) quantify over degrees or individuals (Ahn & Sauerland 2017; Pasternak & Sauerland 2022; Spithas 2022; a.o.).

- (10) a. The fruit supplier sold 60% olives_F (repeated from (2b))
 b. 60%(λd (the fruit supplier sold d -olives_F))

Following Dotlačil & Nouwen (2016) and Li (2022), however, I'll assume, in addition to a quantificational analysis of *percent* (11), that the domain of degrees is pluralized just as entities are. \min_{\subseteq} (12) returns the unique minimum degree from an input predicate of atoms and/or pluralities.

- (11) $\llbracket[\text{percent}]\rrbracket = \lambda d \lambda D. \frac{\min_{\subseteq} D}{\min_{\subseteq}(\text{dom } D)} \geq \frac{d}{100}$ d \rightarrow Sd \rightarrow t
 (12) $\min_{\subseteq} := \lambda D \lambda d. D d \wedge \neg \exists d' [D d' \wedge d' \sqsubset d]$ Sd \rightarrow d
 (13) $\llbracket[\text{size}]\rrbracket := \lambda d \lambda x. |x|^+ \sqsubseteq d$ d \rightarrow Se

To mesh a degree denotation for numerals with their intersective meaning from (8), I'll

³For an atomic type α , $S\alpha$ is simply shorthand for the corresponding set type $\alpha \rightarrow \mathfrak{t}$. Types and meta-language expressions are recorded in **sans serif**. FA is left-associative, and type complexity right-associative by default.

assume a type-shifter from the former to the latter (13). I notate our version as **size**, though it goes under different guises (see [Bylinina & Nouwen 2020](#) for an overview).

We'll also need to revise our original understanding of numerals and gradability:

$$(14) \quad \begin{array}{ll} \text{a. } \llbracket \text{tall} \rrbracket = \lambda d \lambda x. \mu_{\text{height}} x \sqsubseteq d & (\text{Dotlačil \& Nouwen 2016}) \\ \text{b. } \llbracket \text{size} \rrbracket(\llbracket \text{two} \rrbracket)(\llbracket \text{olives} \rrbracket) = \lambda x. \textcircled{\otimes} \text{olive } x \wedge |x|^+ \sqsubseteq 2 & (\text{cf. (8c)}) \end{array}$$

For the degree quantifier *60%*, attempting to compose with a gradable predicate will fail.

The natural solution is for the RM phrase to occupy a higher scope and abstract over the degree argument (15)-(16).⁴ The substance noun is shifted to a gradable denotation with a version of the off-the-shelf measure operator from, e.g., [Rett \(2014\)](#), [Solt \(2015\)](#).

$$(15) \quad \begin{array}{c} \text{MeasP} :: \text{Se} \\ \lambda x. \textcircled{\otimes} \text{olive } x \wedge \mu x \sqsubseteq 2 \\ \swarrow \quad \searrow \\ d \quad \lambda d \lambda x. \textcircled{\otimes} \text{olive } x \wedge \mu x \sqsubseteq d \\ \swarrow \quad \searrow \\ \text{meas} \quad \text{olives}_F \end{array} \quad (16) \quad \begin{array}{c} S' :: t \\ \frac{\min_{\sqsubseteq} \llbracket S \rrbracket}{\min_{\sqsubseteq} (\text{dom} \llbracket S \rrbracket)} \geq \frac{60}{100} \\ \swarrow \quad \searrow \\ 60\% \quad S :: \text{Sd} \\ \lambda d. \exists x [\textcircled{\otimes} \text{olive } x \wedge \\ \mu x \sqsubseteq d \wedge \text{sell } x \text{ Aldi}] \end{array}$$

For count nouns, the contextual measure function μ will amount to the non-negative cardinality function from earlier.⁵ The subject is swapped with a proper name for simplicity. (16) is true just in case ...

$$(17) \quad \frac{\min(\lambda d. \exists x [\textcircled{\otimes} \text{olive } x \wedge |x|^+ \sqsubseteq d \wedge \text{buy } x \text{ Aldi}])}{\min(\text{dom}(\lambda d. \exists x [\textcircled{\otimes} \text{olive } x \wedge |x|^+ \sqsubseteq d \wedge \text{buy } x \text{ Aldi}]))} \geq \frac{60}{100}$$

4. Modification and scopal interactions

We move now to the observation of cumulativity that our machinery has built up to. (18) is true just in case the maximum number of interviewing recruiters is 2, and the maximum proportion of women interviewed by recruiters (out of all interviewees) is 60% (19).

(18) Exactly two recruiters interviewed exactly 60% women_F (between them).

⁴Accessing Roothian focus alternatives to the substance noun will allow for the denominator to make reference to a domain larger than the noun itself. See [Pasternak & Sauerland \(2022:257–261\)](#) for discussion.

⁵Another strategy for measuring cardinality would be to gather the non-negative subparts (if any) first, and then feed them as a set or distinct sum to a non-polarity-specific $|\cdot|$.



The cumulative intuition does not fall out from the assumption that the modified numeral is existential (since (18) is false when there are more than two recruiters).

The challenge is, additionally, how to prevent the sentence from yielding truth when there are multiple different combinations of recruiters each with a 60%-40% split.

We've arrived at 'van Benthem's puzzle' ([van Benthem 1986](#); [Krifka 1999](#); [Brasoveanu 2013](#); [Charlow 2021](#); a.o.). RMs serve as a useful window into this problem, given *vague* quantifiers cannot be used in modified-numeral constructions (20).

(20) *{exactly, up to, at most, ...} many/few recruiters ...

With negation over atoms, we obtain a straightforward understanding of modified numerals — *exactly two recruiters* denotes the set of pluralities with (i) two non-negative atoms and (ii) negative values for all other recruiters:

(21) {Anne ⊕ Brooke ⊕ −Claire, Anne ⊕ −Brooke ⊕ Claire, −Anne ⊕ Brooke ⊕ Claire}

Hence, we get the intuition (22) that *exactly n x's* implies every other *x* didn't participate.

(22) a. A: Exactly two students came to the talk, Ann and Brooke.
b. B: # Who else showed up? *or*
c. B: # Did Claire show up? *or*
d. ...

The modifier introduces a parthood maximality condition on the entities of a gradable predicate and requires the non-negative cardinality of those entities to be of a certain degree.⁶

$$(23) \quad \llbracket \text{exactly} \rrbracket(\llbracket \text{two} \rrbracket) = \lambda A \lambda x. x \in \mathbf{max}_{\sqsubseteq}(A \ 2) \wedge \mathbf{max}_{\leq}(\lambda d. A \ d \ x) \sqsubseteq 2 \quad (d \rightarrow \text{Se}) \rightarrow \text{Se}$$

The operator for scalar maximum selects from atoms (24) ([Heim 2000](#); [Buccola & Spector 2016](#)), so the degree conjunct in the modifier expresses equality.

$$(24) \quad \llbracket \mathbf{max}_{\leq} \rrbracket := \lambda D_{\text{atoms}} t d. D \ d \wedge \forall d' [D \ d' \rightarrow d' \leq d]$$

⁶Elliott (2024) assumes that plural nouns automatically include only the maximal pluralities. By contrast, I'll encode this into the semantics of *exactly*, rather than try to generalize to plural nouns.

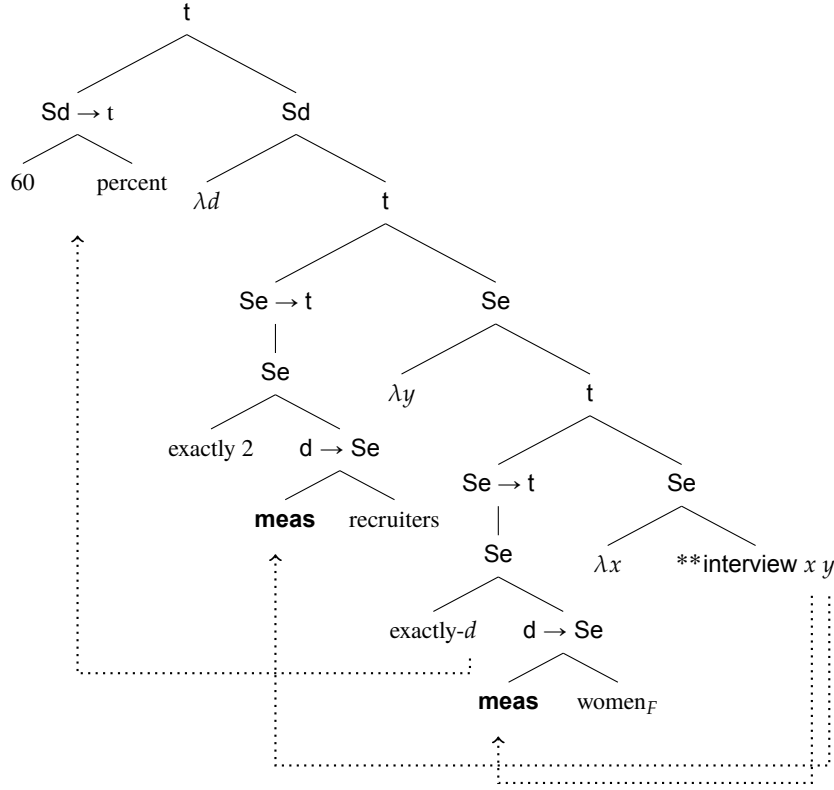


Figure 2: Scoping a modified proportion

Applying (23) to the gradable predicate successfully yields the set of maximal recruiter pluralities of size two:

$$(25) \quad \llbracket \text{exactly} \rrbracket (\llbracket \text{two} \rrbracket) (\llbracket \text{meas recruiters} \rrbracket) \\ = \lambda x. x \in \max_{\sqsubseteq} (\underbrace{\llbracket \text{meas} \rrbracket \text{recruiter } 2}_{\lambda y. \text{recruiter } y \wedge \mu y \sqsubseteq 2}) \wedge \max_{\leq} (\lambda d. \underbrace{\llbracket \text{meas} \rrbracket \text{recruiter } d}_{\text{recruiter } x \wedge \mu x \sqsubseteq d}) \sqsubseteq 2$$

We can extend the same idea to the modified proportion, where the size restriction of the pluralities is abstracted over by the degree quantifier:

$$(26) \quad 60\% \lambda d [\dots \text{ex. two recruiters} [\dots \text{ex. } d \text{ women}_F]]$$

The resulting structure is Figure 2, where the arguments are lifted to scope-takers (27a).

E (27b) is the canonical type-shifter from predicates to quantifiers introducing existential force (following Partee 1987).

$$(27) \quad \text{a. } \mathbf{E}(\llbracket \text{exactly two recruiters} \rrbracket) = \lambda Q. \exists x [\llbracket (25) \rrbracket x \wedge Q x] \\ \text{b. } \mathbf{E} := \lambda P \lambda Q. \exists x [P x \wedge Q x]. \quad \text{Se} \rightarrow \text{Se} \rightarrow t$$

** is the relational cumulation operator (Sternefeld 1998; Beck & Sauerland 2000; Dotlačil & Nouwen 2016) (28), but I'll assume a polarized version (see Elliott 2024) such that no negative atoms participate.

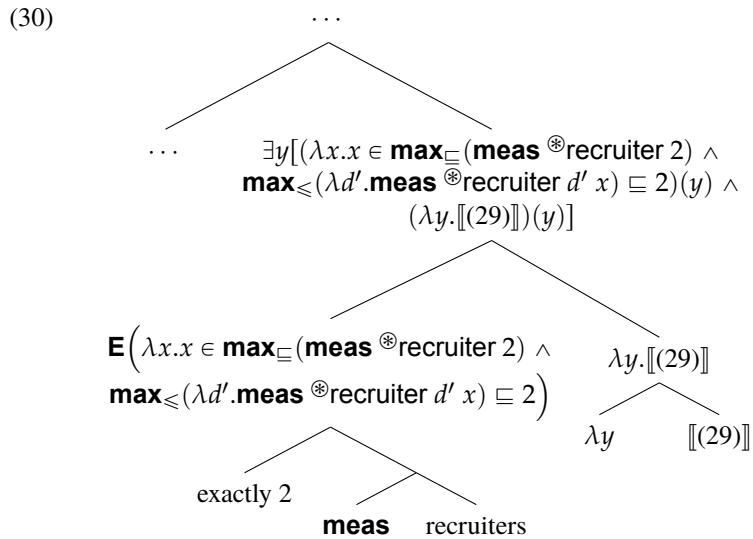
$$(28) \quad \mathbf{**}R = \\ \mathbf{min}_{\sqsubseteq} \{R' \mid R \subseteq R' \wedge \forall x, x', y, y' [\langle x, x' \rangle, \langle y, y' \rangle \in \mathbf{**}R \rightarrow \langle x \oplus y, x' \oplus y' \rangle \in \mathbf{**}R]\}$$

The bottom of the LF corresponding to Figure 2, for instance, is given in (29) below. Roughly, we obtain truth just in case there exists a maximal plurality of women of non-negative size d that was interviewed by some y .

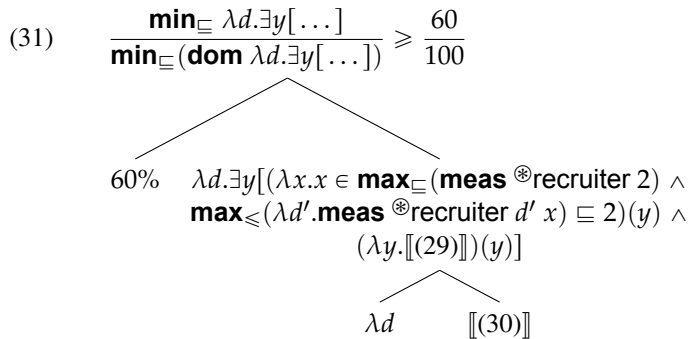
$$(29) \quad \dots \\ \dots \quad \exists x [(\lambda x. x \in \max_{\sqsubseteq} (\text{meas} \textcircled{*} \text{woman } d) \wedge \\ \max_{\leq} (\lambda d'. \text{meas} \textcircled{*} \text{woman } d' x) \sqsubseteq d)(x) \wedge \\ (\lambda x. \mathbf{**} \text{interview } x y)(x)] \\ \mathbf{E} \left(\lambda x. x \in \max_{\sqsubseteq} (\text{meas} \textcircled{*} \text{woman } d) \wedge \lambda x. \mathbf{**} \text{interview } x y \right. \\ \left. \max_{\leq} (\lambda d'. \text{meas} \textcircled{*} \text{woman } d' x) \sqsubseteq d \right) \\ \text{exactly-}d \quad \text{meas} \quad \text{women}_F$$

The y variable is bound by the *recruiters* quantifier, and the d variable by *60%*.

The resulting meaning at the level of *recruiters* quantifier will yield truth just in case there exists a maximal plurality y of recruiters of (non-negative) size 2, and y interviewed exactly d women_F (i.e., (29)).



The RM phrase takes the degree predicate of (30) as its argument (31).



With modifiers that aren't non-monotone, such as (32a), we likewise gather an accurate result.

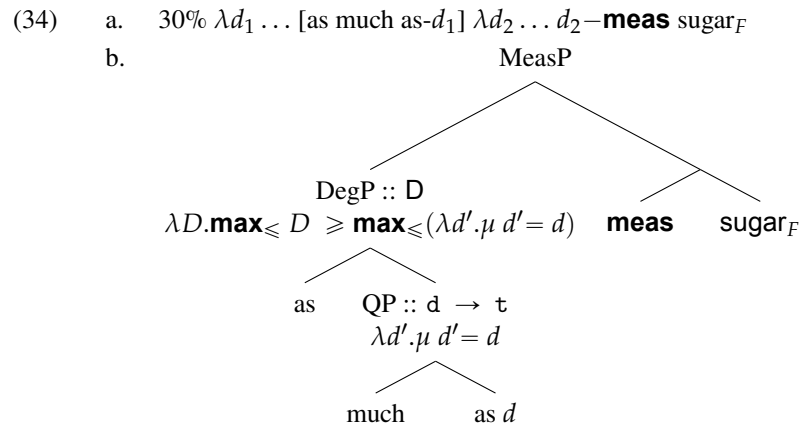
- (32) a. Fanta contains as much as 40% sugar_F.
b. 40%(λd (Fanta contains as much as d-meas sugar_F)).

In terms of the semantics, the modification in (32a) with the equative will yield the same truth conditions as the measure phrase without modification.

Following Rett (2010, 2015, 2020), the equative operator is quantificational, as in (33).

- (33) Adapted from Rett (2015:447–449)
a. Danny Gibbons is as tall as 6'3".
 $\rightsquigarrow \mathbf{max}_{\leq}(\lambda d.\text{tall}(\text{Danny})(d)) \geq 6'3''$
 $\rightsquigarrow \mathbf{max}_{\leq}(\lambda d.\text{tall}(\text{Danny})(d)) = 6'3''$ (implicature strengthening)
b. $\llbracket \text{as} \rrbracket = \lambda D' \lambda D. \mathbf{max}_{\leq} D \geq \mathbf{max}_{\leq} D'$

With the RM phrase still undergoing QR, the degree variable is vacuously enforced as the lower bound to the equative operator. Both operators can take scope (34). The denotation for the quantity adjective is pulled from Coppock & Bogal-Allbritten (2018).



5. Narrow scope and degree pluralities

The empirical facts for RM constructions differ cross-linguistically. In Korean, non-conservative RMs may be adnominal floating quantifiers (Ahn & Ko 2022), and, in German and Bulgarian, they may rely on structural height instead of focus (Gehrke & Wągiel 2023).

A crucial observation for our purposes is that, in Mandarin, RM phrases are observed to uniformly scope under negation and intensional operators (Li 2022), as in (35)-(36).⁷

- (35) Huawei meiyou gu [70% de bendi-ren_F].
Huawei not hire 70% DE local-people
a. 'It's not the case that 70% of Huawei's employees were locals'
b. #'70% of the people that Huawei didn't hire were locals'

(Li 2022:7,21)

⁷Li (2022) does not report data on any scopal facts for modified RMs in Mandarin, instead only for bare RMs.

- (36) Huawei xiang zhao [sanfenzhiyi de ruanjian_F gongchengshi].
 Huawei want recruit one.third DE software engineers]
- a. ‘For all worlds w compatible with Huawei’s desire in the actual world w_0 , Huawei recruiters one-third software_F engineers in w .’
- b. #‘For a group of people X such that, for all worlds w compatible with Huawei’s desire in the actual world w_0 , Huawei recruits X in w_0 . One-third of X are software_F engineers.’

(Li 2022:23)

Indeed, this is also the case for English (37).

- (37) A company has a quota requiring some of its workforce to be locally sourced.
- a. The company didn’t hire 30% locals_F. $\neg > \%$
- b. The company must hire 30% locals_F. $\square > \%$
- c. The company might’ve hired 30% locals_F. $\diamond > \%$

I’ll follow a solution for narrow scope that leverages the fact that division over degrees is undefined for pluralities.

For (36b), the RM phrase taking scope over the intensional universal quantifier *want* means the resulting numerator is the minimum degree plurality *across possible worlds* (Li 2022):

$$(38) \frac{\min_{\sqsubseteq}(\lambda d. \llbracket \text{Huawei wants to recruit } d\text{-meas software}_F \text{ engineers} \rrbracket)}{\min_{\sqsubseteq}(\text{dom}(\lambda d. \llbracket \text{Huawei wants to recruit } \dots \rrbracket))} \geq \frac{1}{3}$$

If the size of hired recruiter pluralities differs from one world to the next, the parthood minimum will itself be a plurality:

$$(39) \frac{d_{w_1} \oplus d_{w_2} \oplus d_{w_3} \oplus \dots}{\dots} \geq \frac{1}{3}$$

Hence, the RM expression must take narrow scope. To enforce this, I’ll first follow Buccola & Spector (2016), who assume that modified numerals denote intensional operators, by revising our denotation of *percent* to instead compose with the intension of a degree predicate.

Intensional FA (IFA) proceeds as in (40), adapted from Buccola & Spector (2016:183).

$$(40) \begin{array}{c} \alpha :: \tau \\ \llbracket \beta \rrbracket^w(\lambda w'. \llbracket \gamma \rrbracket^{w'}) \\ \swarrow \quad \searrow \\ \beta :: (s \rightarrow \sigma) \rightarrow \tau \quad \gamma :: \sigma \end{array}$$

Definitions are otherwise preserved, so long as the world of evaluation is parameterized:

$$(41) \llbracket \text{two meas recruiters} \rrbracket^w = \lambda x. \textcircled{\text{recruiter}} w x \wedge \mu x \sqsubseteq 2$$

To ensure a meaning still obtains, but only with narrow scope, *percent* denotes a function of type $\mathbf{d} \rightarrow (\mathbf{s} \rightarrow \mathbf{Sd}) \rightarrow \mathbf{t}$, defined only over atomic degrees across possible worlds (42).

The partial operator ∂ is used to allow for straightforward multi-part presuppositions (Coppock & Beaver 2015; Coppock 2022; a.o.), since we’ll need to quantify over worlds and apply them to the predicate in our revised denotation:

$$(42) \llbracket 60 \text{ percent} \rrbracket^w = \lambda D. \partial(\forall w' [\text{atom}(\min_{\sqsubseteq}(D w'))]) \wedge \frac{\min_{\sqsubseteq}(D w)}{\min_{\sqsubseteq}(\text{dom}(D w))} \geq \frac{60}{100}$$

There are two discussion items I’ll set aside for another time.

- Atomic negation for marking degrees too?

We may consider the possibility of polarizing degrees in a similar way to entities. Some first-passes would be via corrective-*but* (43a)-(43b) or *neither*-expressions (43c).

- (43) a. The company hired not 40 but 50% women_F.
 b. Anne is as tall as not 5 but 6 feet.
 c. Neither 5 nor 10 students passed the exam.

The hurdle for this would be figuring out the non-monotonic interpretation of (at least) the first of the two degrees. Moreover, if we say atomic negation *isn’t* at play, the assumption in the literature that *6 feet* denotes a degree (recall also (33)) may need to be revised for (43b).

- Hesitations for degree quantifiers scoping over generalized quantifiers.

It’s clear that our assumptions for *percent* seemed to have set aside the Heim-Kennedy Generalization (HKG) — i.e., that degree quantifiers cannot outscope e-type quantifiers (Kennedy 1997; Heim 2000).

In response to this objection, I believe it’s worth noting that this concern crucially depends on what definition of the HKG we’re considering. Our working analysis *not* violate, for instance, the HKG variant suggested by Crnič (2017) (44a), if we liken *exactly* to Crnič’s SHIFT operator defined in (44b).

- (44) a. Modified HKG: If the scope of an e-type quantifier contains the trace d of a degree quantifier, d must be an argument to SHIFT.
 b. $\llbracket \text{SHIFT} \rrbracket := \lambda d \lambda A \lambda x. \text{max}_{\leq}(\lambda d'. A d' x) \sqsubseteq d \quad \mathbf{d} \rightarrow (\mathbf{d} \rightarrow \mathbf{Se}) \rightarrow \mathbf{Se}$

6. Recap

In this talk, we've taken a first stab at the meaning and composition of modified proportions. To provide an account, a degree-quantificational semantics for non-conservative relative measurement (Pasternak & Sauerland 2022; Li 2022) is combined with an approach to pluralities using atomic negation over individuals (Bledin 2024; Elliott 2024).

Acknowledgements

This talk is a revised and extended version of Kato (2024), which has greatly improved from discussion with Patrick Elliott. For feedback on one or more of the ideas presented here, I'm grateful to Pranav Anand, Dylan Bumford, Jess Law, Haoze Li, and Maziar Toosarvandani.

References

- Ahn, Dorothy & Heejeong Ko (2022). On non-conservativity of Korean floating quantifiers. *Glossa: A Journal of General Linguistics* 7, 1–34.
- Ahn, Dorothy & Uli Sauerland (2015a). The grammar of relative measurement. D'Antonio, Sarah, Mary Moroney & Carol Rose Little (eds.), *Proceedings of SALT 25*, 125–142.
- Ahn, Dorothy & Uli Sauerland (2015b). Non-conservative quantification with proportional quantifiers: Crosslinguistic data. Bui, Thuy & Deniz Özyıldız (eds.), *Proceedings of NELS 45*, 1–11.
- Ahn, Dorothy & Uli Sauerland (2017). Measure constructions with relative measurements: Towards a syntax of non-conservative construals. *The Linguistic Review* 34, 215–248.
- Beck, Sigrid & Uli Sauerland (2000). Cumulation is needed: A reply to Winter (2000). *Natural Language Semantics* 8, 349–371.
- van Benthem, Johan (1986). *Essays in logical semantics*. Dordrecht: Springer.
- Bledin, Justin (2024). Composing menus. Manuscript, Johns Hopkins University.
- Brasoveanu, Adrian (2013). Modified numerals as post-suppositions. *Journal of Semantics* 30, 155–209.
- Buccola, Brian & Benjamin Spector (2016). Modified numerals and maximality. *Linguistics & Philosophy* 39, 151–199.
- Bylina, Lisa & Rick Nouwen (2020). Numeral semantics. *Language and Linguistics Compass* 14, 1–18.
- Champollion, Lucas & Manfred Krifka (2016). Mereology. Aloni, Maria & Paul Dekker (eds.), *The Cambridge handbook of formal semantics*, Cambridge: Cambridge University Press, 369–388.
- Charlow, Simon (2021). Post-suppositions and semantic theory. Manuscript to appear in *Journal of Semantics*.
- Coppock, Elizabeth (2022). Part-introducing *percent* in English. *Glossa: A Journal of General Linguistics* 7, 1–38.
- Coppock, Elizabeth & David Beaver (2015). Definiteness and determinacy. *Linguistics & Philosophy* 38, 377–435.
- Coppock, Elizabeth & Elizabeth Bogal-Allbritten (2018). Forces at the interface of gradability and quantification. Maspong, Sireemas, Brynhildur Stefánsdóttir, Katherine Blake & Forrest Davis (eds.), *Proceedings of SALT 28*, 747–767.
- Crnič, Luka (2017). More on *less*. Lamont, Andrew & Katerina Tetzloff (eds.), *Proceedings of NELS 47*, Amherst, MA: GLSA, 215–229.
- Dotlačil, Jakub & Rick Nouwen (2016). The comparative and degree pluralities. *Natural Language Semantics* 24, 45–78.
- Elliott, Patrick D. (2020). A plea for equality: Remarks on Moltmann's semantics for clausal embedding. *Theoretical Linguistics* 46, 241–251.
- Elliott, Patrick D. (2024). Mixed-polarity pluralities: A solution to van Benthem's problem. Paper presented at Sinn und Bedeutung 29 (Noto).
- Fine, Kit (2017). A theory of truthmaker content I: Conjunction, disjunction and negation. *Journal of Philosophical Logic* 46, 625–674.
- Gehrke, Berit & Marcin Wągiel (2023). On word order and non-conservative percentage quantification in Slavic and German. *Glossa: A Journal of General Linguistics* 8, 1–48.
- Heim, Irene (2000). Degree operators and scope. Brendan, Jackson & Tanya Matthews (eds.), *Proceedings of SALT X*, Ithaca, NY: Cornell University, 40–64.
- Herburger, Elena (1997). Focus and weak noun phrases. *Natural Language Semantics* 5, 53–78.
- Kato, Andrew (2024). Relative quantification and equative scope-taking. Carcassi, Fausto, Tamar Johnson, Søren Brinck Knudstorp, Sabrina Domínguez Parrado, Pablo Rivas Robledo & Giorgio Sbardolini (eds.), *Proceedings of the 24th Amsterdam Colloquium*, Amsterdam: Institute for Logic, Language and Computation, 203–210.
- Kennedy, Christopher (1997). *Projecting the adjective*. Ph.D. thesis, University of California, Santa Cruz.

- Krifka, Manfred (1999). At least some determiners aren't determiners. Turner, Ken (ed.), *The semantics/pragmatics interface from different points of view*, vol. 1, Amsterdam: Elsevier, 257–291.
- Li, Haoze (2022). Relative measurement and scope in Mandarin. *Glossa: A Journal of General Linguistics* 7, 1–40.
- Link, Godehard (1983). The logical analysis of plurals and mass terms. *Meaning, use, and interpretation in language*, Berlin: De Gruyter, 303–323.
- Moltmann, Friederike (2020). Truthmaker semantics for natural language: Attitude verbs, modals, and intensional transitive verbs. *Theoretical Linguistics* 46, 159–200.
- Partee, Barbara (1987). Noun phrase interpretation and type-shifting principles. Groenendijk, Jeroen, Dick de Jongh & Martin Stokhof (eds.), *Studies in discourse representation theories and the theory of generalized quantifiers*, Dordrecht: Foris, 115–144.
- Partee, Barbara (1989). Many quantifiers. Powers, Joyce & Kenneth de Jong (eds.), *ESCOL 89: Proceedings of the Eastern States Conference on Linguistics*, Ohio State University, 383–402.
- Pasternak, Robert & Uli Sauerland (2022). German measurement structures: Case-marking and non-conservativity. *Journal of Germanic Linguistics* 25, 221–272.
- Rett, Jessica (2010). Equatives, measure phrases, and NPIs. Aloni, Maria, Harald Bastiaanse, Tikitou de Jager & Katrin Schulz (eds.), *Logic, language, and meaning: 17th Amsterdam Colloquium, Amsterdam, The Netherlands, December 16-18, 2009, revised selected papers*, Berlin: Springer, 364–373.
- Rett, Jessica (2014). The polysemy of measurement. *Lingua* 143, 242–266.
- Rett, Jessica (2015). Measure phrase equatives and modified numerals. *Journal of Semantics* 32, 425–475.
- Rett, Jessica (2020). Separate but equal: A typology of equative constructions. Hallman, Peter (ed.), *Interactions of degree and quantification*, Brill, 163–204.
- Solt, Stephanie (2015). Q-adjectives and the semantics of quantity. *Journal of Semantics* 32, 221–273.
- Spathas, Giorgios (2022). Domain restricted measure functions and the extent readings of relative measure. Gotzner, Nicole & Uli Sauerland (eds.), *Measurements, numerals and scales: Essays in honour of Stephanie Solt*, Palgrave Macmillan, 275–288.
- Sternefeld, Wolfgang (1998). Reciprocity and cumulative predication. *Natural Language Semantics* 6, 303–337.
- Westerståhl, Dag (1985). Logical constraints in quantifier languages. *Linguistics and Philosophy* 8, 387–413.