Modifying degrees and their proportions

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Setting the stage

Precise proportional expressions (‘relative measures’; RMs) can occur with restrictors that are unexpected when just reading off the overt structure.

(1) For their new cohort . . .

a. The department admitted 60% semanticists$_F$,

b. (and 40% phonologists$_F$.)

Setting the stage

That is, when I utter (1a), I'm intuitively making an assertion about the *total* group of people some department admitted (restrictor) — that, of which, 60% are semanticists (scope).

(1) For their new cohort . . .

a. The department admitted $\boxed{\text{60\%}}$ semanticists$_F$,
b. (and 40% phonologists$_F$.)

Setting the stage

And, we know that we aren't dealing with an adverbial modifier:

(2) a. The dept. admitted {mostly, 60%} semanticists$_F$.
b. The dept. {mostly, *60%} admitted semanticists$_F$.
c. The dept. admitted semanticists$_F$, {mostly, *60%}.

(data varies between languages, e.g., Ahn & Ko 2022; Kim 2024 on Korean)

Setting the stage

This sort of observation has been known since Sauerland (2014), and leaves us with a pattern reminiscient of the ‘relative proportional’ reading of the vague quantifiers (3).
(see also Ahn & Sauerland 2015a,b, 2017)

(3) a. Many [$_S$ Scandinavians] [$_N$ have won the NPL].
$\rightsquigarrow |S \cap N| \geqslant \theta_c$ (cardinal)
$\rightsquigarrow {}^{|S \cap N|}/_{|S|} \geqslant \theta_c$ (proportional)
b. Many [$_S$ Scandinavians$_F$] [$_N$ have won the NPL].
$\rightsquigarrow {}^{|S \cap N|}/_{|N|} \geqslant \theta_c$ (relative proportional)

(Westerståhl 1985:403; Partee 1989; Herburger 1997; a.o.)

Setting the stage

Yet, unlike *many*/*few*, we can add typical degree modifiers onto RMs (4), even when they have this surprising scope.

(4) The dept. admitted {at least, . . . } 60% semanticists$_F$.

Setting the stage

In this respect, we arrive at a pattern similar to modified numerals — which also includes arriving at similar problems.

(5) Exactly three boys watched exactly five movies.
(Brasoveanu 2013:155)

(6) Exactly two recruiters interviewed exactly 60% women$_F$.

Setting the stage

Under a cumulative reading, the classic example (5) is true just in case the total number of movie-seeing boys is 3 and the total number of seen movies is 5.

(5) Exactly three boys watched exactly five movies.
(Brasoveanu 2013:155)

Such examples represent a distinct compositional challenge, which is compounded with the separate problem of non-monotonic modifiers (e.g., Benthem 1986).

The big picture

So, our goal is to provide an account for RM phrases that extends to their semantics under modification.

I'll proceed by setting up RMs as scope-takers to capture the basic facts at the outset.

To avoid pseudo-cumulativity and van Benthem's problem, I'll upgrade our degree semantics with the ability to measure pluralities that encode information about non-participating alternatives. (Bledin 2024; Elliott 2025)

Interpreting proportions

We can draw an immediate parallel between RMs and . . .

- Relative-proportional *many*/*few*
- Relative superlative constructions

Interpreting proportions

Namely, the Heim (1999) approach to relative superlatives and Romero's (2018, 2021) degree semantics for *many*+POS:

(7) a. Many Scandinavians$_F$ have won the NPL.
b. John$_F$ climbed the tallest mountain (among all the mountain climbers).

Interpreting proportions

That is, RM nouns such as *percent* can take the form of scope-takers that abstract over degrees, and compare alternative abstractions-over-degrees via semantic focus.

(see also Li 2022 on Mandarin and Pasternak & Sauerland 2022 on German)

A basic semantics

After affording ourselves a typical scalar-maximum operator and a way to convert predicates into gradable denotations . . .
(Heim 2000; Rett 2014; Solt 2015)

(8) a. $\mathtt{max}_{\leqslant} \coloneqq \lambda D.\iota d.D\,d \land \neg\exists d'.D\,d' \land d' \geqslant d \;::\; (\mathtt{d} \to \mathtt{t}) \to \mathtt{d}$

b. $\mathtt{meas} \coloneqq \lambda Pdx.P\,x \land \mu_c\,x \geqslant d \;::\; (\mathtt{e} \to \mathtt{t}) \to \mathtt{d} \to \mathtt{e} \to \mathtt{t}$

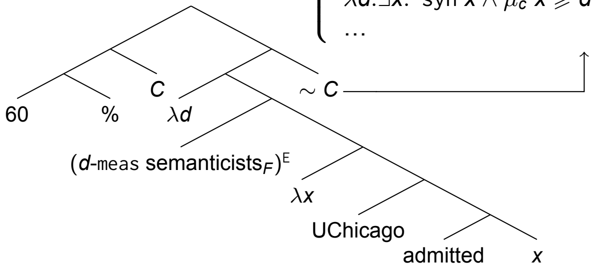
We can say that a scope-taking proportion measures a clause's scalar maximum against the maxima of its alternatives:

(9) $$[\![\text{percent}]\!]^c \coloneqq \lambda dCD.\frac{\mathtt{max}_{\leqslant}\,D}{\mathtt{sum}\,[\mathtt{max}_{\leqslant}\,D' \mid D' \in C]} \geqslant \frac{d}{100}$$

$$::= \mathtt{d} \to \{\mathtt{d} \to \mathtt{t}\} \to (\mathtt{d} \to \mathtt{t}) \to \mathtt{t}$$

A representative sketch

$$\left\{\begin{array}{l} \lambda d.\exists x.{}^{*}\mathtt{sem}\ x \wedge \mu_c\ x \geqslant d\ \wedge \mathtt{admit}\ x\ \mathtt{UChicago}, \\ \lambda d.\exists x.{}^{*}\mathtt{phon}\ x \wedge \mu_c\ x \geqslant d\ \wedge \mathtt{admit}\ x\ \mathtt{UChicago}, \\ \lambda d.\exists x.{}^{*}\mathtt{syn}\ x \wedge \mu_c\ x \geqslant d\ \wedge \mathtt{admit}\ x\ \mathtt{UChicago}, \\ \ldots \end{array}\right\}$$



$\cdot^{\mathsf{E}}$ is the canonical existential type-shifter (Partee 1987):

(10) $\cdot^{\mathsf{E}} := \lambda PQ.\exists x.P\ x \wedge Q\ x$

A representative sketch

The corresponding denotation:

(11) $[\![\text{UChicago admitted 60\% semanticists}_F]\!]^c =$

$$\frac{\mathtt{max}_{\leqslant}\ \lambda d.\exists x.{}^{*}\mathtt{sem} \wedge \mu_c\ x \geqslant d \wedge \mathtt{admit}\ x\ \mathtt{UChicago}}{\mathtt{sum}\ [\mathtt{max}_{\leqslant}\ D' \mid D' \in C]} \geqslant \frac{60}{100}$$

Onto modifiers: van Benthem's puzzle

What about degree modifiers for RMs? Well, we know since at least van Benthem (1986:52–53) that we lose non-increasing measurement under existential raising.

If we naively assumed *exactly two semanticists* denotes the property in (12), we admit too weak a quantificational meaning:

(12) $(\lambda x.{}^{*}\mathtt{sem}\; x \wedge \mu_{\boldsymbol{c}}\; x = 2)^{\mathsf{E}} = \lambda P.\exists x.{}^{*}\mathtt{sem}\; x \wedge \mu_{\boldsymbol{c}}\; x = 2 \wedge P\; x$

(13) $\mathtt{smokes}\; \boxed{a \oplus b} \oplus c \oplus \;\ldots\; = \top$

$\downarrow$ verifies

$\exists x.{}^{*}\mathtt{sem}\; x \wedge \mu_{\boldsymbol{c}}\; x = 2 \wedge \mathtt{smokes}\; x$

Onto modifiers: Maximality

So, a revised denotation might enforce maximality with respect to the predicate (semanticists, in our case).

But, if we have multiple modified numerals, each maximality condition will have to be scoped with respect to one another.
(Brasoveanu 2013; Charlow 2021; a.o.)

Onto modifiers: Maximality

This problematically admits an unattested 'pseudo-cumulative' reading for an example like (14):

(14) Exactly three boys saw exactly five movies.
(Brasoveanu 2013:155)

. . . that there can be *multiple* three-boy pluralities that between them saw five movies (with surface-scope QR ordering).

Modifiers for RMs

Since RMs can be modified, a similar problem will arise for us.

And, we already know that non-degree quantificational proportions can yield cumulative readings:

(15) In Guatemala, (at most) 3% of the population owns (at least) 70% of the land. (Krifka 1999:262)

Modifiers for RMs

Indeed, we can devise the same version for the constructions we've looked at thus far:

(16) Exactly two recruiters interviewed exactly 60% women$_F$.

Under a cumulative reading, (16) says nothing about the proportions of interviewees for *each* recruiter (that would be distributive), only about the resulting total between them.

Modifiers for RMs

Indeed, we can devise the same version for the constructions we've looked at thus far:

(16) Exactly two recruiters interviewed exactly 60% women$_F$.

For instance, (16) would be a valid answer to (17).

(17) The boss asks you (the logistics person): 'What was our total turnout of recruiters, and what was the total demographic makeup of interviewees?'

Modifiers for RMs

And a valid situation verifying (18) would be one such as (19).

(18) Exactly three universities admitted exactly 50% semanticists$_F$ (between them).

(19) a. Admitting departments: [UIUC, UChicago, NU]
b. Admitted students: $[s_1, s_2, p_1, p_2]$

Modifiers for RMs

One possible solution to prevent pseudo-cumulativity for modified RMs is to revise our semantics from the ground up with dynamic ‘post-suppositions’ (Brasoveanu 2013) that are simultaneously evaluated last (‘pseudo-wide scope’).

I’ll instead opt to upgrade our original system from earlier.

Negative entities

Ongoing work by Bledin (2024) argues for a domain of individuals that includes ‘negative entities’, in the context of a truthmaker semantics for coordination (e.g., *Anne and nobody else, not Anne but Mary*).

Elliott (2025) extends the main theoretical idea for, among other things, numerals and a GQs-as-sets system.

Negative entities

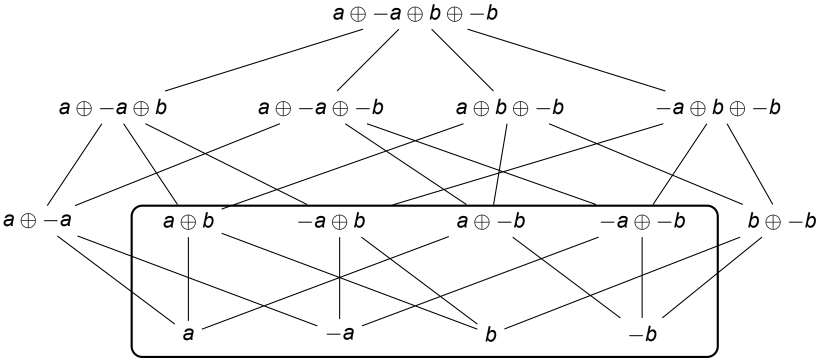
Suppose that every atomic individual x in D_e has a marked counterpart $-x$. We also take inspiration from Krifka's (1999) suggestion of 'polarity-marked alternatives' with focus.

(20) a. $^*P = \mathtt{min}\{P' \mid P \subseteq P' \wedge \forall x, y \in P'.x \oplus y \in P'\}$
b. $^\circ P = {}^*P \setminus \{X \in {}^*P \mid \exists x \sqsubseteq X. -x \sqsubseteq X\}$

For a revised summation operator $^\circ\cdot$ for a given predicate, we obtain the smallest sum-closed superset, minus contradictory combinations (e.g., $x \oplus -x$).

(just one way to define Link's (1983) $^*\cdot$, see Champollion & Krifka 2016)

Negative entities



Predicates will thus lack unique parthood maxima.

Enriched pluralities

It follows that the corresponding denotations of numerals will include, negative, non-negative, and mixed pluralities.

(21) a. $[\![\text{one meas semanticist}]\!]^c = \lambda x.{}^\circ\texttt{sem}\ x \wedge \mu_c\ x \geqslant 1$
$\rightsquigarrow \{a, b, c, a \oplus b, a \oplus -b, \ldots\}$

b. $[\![\text{two meas semanticists}]\!]^c = \lambda x.{}^\circ\texttt{sem}\ x \wedge \mu_c\ x \geqslant 2$
$\rightsquigarrow \{a \oplus b, \ldots, a \oplus b \oplus -c, \ldots\}$

- $\mu \mapsto |\cdot|^+$ is shorthand for counting non-negative atoms
- Each class of atoms can be accessed: $(\cdot)^+$ and $(\cdot)^-$
- $X = \{a \oplus -b \oplus -c\} \rightsquigarrow X^+ = \{a\}$
- $X = \{a \oplus -b \oplus -c\} \rightsquigarrow X^- = \{b \oplus c\}$

Modifiers

Now, modifiers can be understood as introducing parthood- and scalar-maximality conditions:

(22) $$[\![\text{exactly}]\!] = \lambda dAx.\mathtt{max}_{\sqsubseteq}(A\ d)(x) \wedge \mathtt{max}_{\leqslant}(\lambda d'.A\ d'\ x) = d$$
$$:: \mathtt{d} \to (\mathtt{d} \to \mathtt{e} \to \mathtt{t}) \to \mathtt{e} \to \mathtt{t}$$

... where the operator for parthood maxima yields a set:

(23) $$\text{max}_{\sqsubseteq} := \lambda Px.P\ x \wedge \neg\exists y.P\ y \wedge y \sqsubset x$$

Modifiers

(24) $\lambda x.\mathtt{max}_{\sqsubseteq}(\lambda y.{}^{\circ}\mathtt{sem}\ y \wedge \mu_c\ y \geqslant 2)(x) \wedge$
$\mathtt{max}_{\leqslant}(\lambda d'.{}^{\circ}\mathtt{sem}\ x \wedge \mu_c\ x \geqslant d') = 2$

exactly 2 $\lambda dx.{}^{\circ}\mathtt{sem} \wedge \mu_c \geqslant d$

$\mathtt{meas}$ semanticists

We obtain the set of parthood-maximal individuals of size d relative to a gradable predicate A.

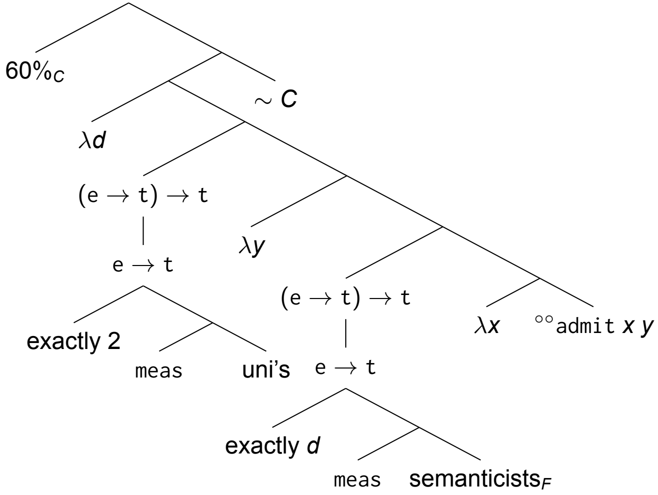
Modifiers and van Benthem's puzzle

(25)

- $\lambda P.\exists x.P\,x \wedge \mathtt{max}_{\sqsubseteq}(\lambda y.{}^{\circ}\mathtt{sem}\;y \wedge \mu_c\;y \geqslant 2)(x) \wedge \mathtt{max}_{\leqslant}(\lambda d'.{}^{\circ}\mathtt{sem}\;x \wedge \mu_c\;x \geqslant d') = 2$
  - E
  - $\lambda x.\mathtt{max}_{\sqsubseteq}(\lambda y.{}^{\circ}\mathtt{sem}\;y \wedge \mu_c\;y \geqslant 2)(x) \wedge \mathtt{max}_{\leqslant}(\lambda d'.{}^{\circ}\mathtt{sem}\;x \wedge \mu_c\;x \geqslant d') = 2$
    - exactly 2
    - $\lambda dx.{}^{\circ}\mathtt{sem} \wedge \mu_c \geqslant d$
      - $\mathtt{meas}$
      - semanticists
- λx $\ldots$

Scoping multiple modifiers

(26)



Cumulation

Since these negative entities propagate through the LF, we'll need a way to compose them with verbs (i.e., otherwise, what would `smoke` $a \oplus -b$ mean?).

As an example, we can use a revised form of the typical cumulativity operator ** (Sternefeld 1998; Beck & Sauerland 2000; a.o.):

(27) $$\begin{aligned} [\![{}^{\circ\circ}R\ X\ Y]\!] = \; & \forall y \in Y^{+}.\exists x \in X^{+}.R\ x\ y \wedge \\ & \forall x' \in X^{+}.\exists y' \in Y^{+}.R\ x'\ y' \wedge \\ & \neg\exists n \in X^{-}.R\ n\ y \wedge \\ & \neg\exists n' \in X^{-}.R\ x\ n' \end{aligned}$$

That is, none of the individuals associated with a negative member participate in the relation. (cf. Elliott 2025 for differences)

Recap

So, we’ve addressed here a few patterns with precise proportional quantifiers (RMs):

- Novel data for RMs with non-monotonic modifiers
- Cumulativity and van Benthem’s problem
- Degree semantics with polarity-marked pluralities

For another time

Are there interesting scopal restrictions that go beyond what we've discussed here?

It's attested for Mandarin that RMs obligatorily take narrow scope with respect to intensional operators/negation (Li 2022).

Certainly, the natural readings for English equivalents follow suit, but perhaps certain contexts could permit wide scope:

(28) a. The university must accept 60% transfer$_F$ students.
b. The university didn't accept 60% transfer$_F$ students.

(29) The university rejected 60% transfers$_F$ and 40% non-transfers. . . . (28b).

Sel. ref. (email: `anmkato@ucsc.edu`)

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